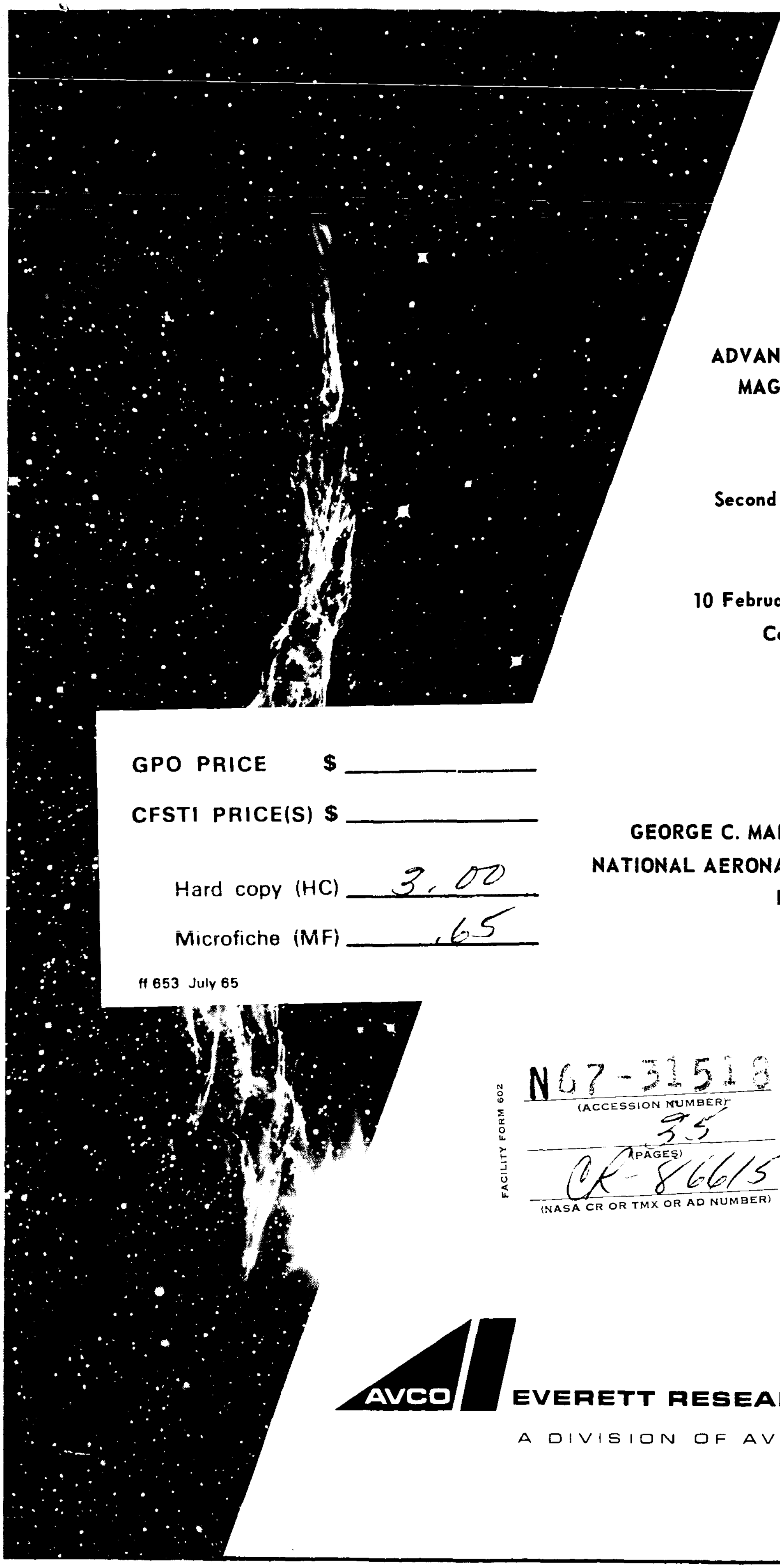

ADVANCED SUPERCONDUCTING MAGNETS INVESTIGATION

Second Quarterly Progress Report

10 February 1967 through 10 May 1967

Contract NAS 8-21037

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AVCO EVERETT RESEARCH LABORATORY

A DIVISION OF AVCO CORPORATION

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Everett, Massachusetts

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I. INTRODUCTION

During the last quarter, analytical work has been concentrated on the problem of modeling the effects of the non-linear heat transfer characteristics of liquid helium on composite conductor stability. In the first section, a first approximation to the effective boiling curve is described and a zero-dimensional analysis is presented which yields a method for determining a stability map for a given superconductor and cooling configuration. Section II indicates some preliminary results of an extension to the one-dimensional theory which appeared in the last progress report (Ref. 1). Section III presents the initial steps toward a solution of a two-dimensional problem with anisotropic thermal conductivity and non-linear heat transfer coefficient.

In addition, experiments have been carried out on test samples of composite conductor with a localized heat source. Construction was such as to allow thermal gradients primarily with length. This configuration resembled the model utilized in the one-dimensional theory and provided data which correlates qualitatively with the behavior predicted by that theory. Typical experimental results are included, but data reduction is not complete as of this writing.

II. CHARACTERISTICS OF A COMPOSITE SUPERCONDUCTOR NEGLECTING GRADIENTS ALONG THE CONDUCTOR, TAKING INTO ACCOUNT A NONLINEAR HEAT TRANSFER CHARACTERISTIC

The case of a superconductor surrounded by a normal substrate which, in turn, is exposed to the liquid helium will be treated taking into account to a first approximation the non-linear heat transfer characteristics of liquid helium.

The superconductor is assumed to be thin enough and to be in excellent thermal and electrical contact with the normal substrate of high thermal conductivity so that the temperature and the voltage is the same in the superconductor and the normal substrate at any point of the conductor cross-section. These assumptions also imply that temperature and voltage gradients within the conductor may be neglected.

The non-linear characteristic of the boiling curve of liquid helium can, to a first approximation, be represented by a region of constant heat transfer coefficient up to a surface temperature T_m which is the maximum temperature at which nucleate boiling can occur; above this temperature a transition to film boiling will occur, and as a first approximation we can assume a constant heat flux per unit area q_m. This is justified by the fact that in a coil magnetic field gradients exist so that adjacent turns will be at different conditions. In such a situation if the surface goes into the film boiling region locally, conduction to adjacent turns will result and a plot of local heat flux per unit area (based only on the area of the conductor in question) versus local temperature will exhibit a characteristic which can be approximated as a constant heat flux per unit area of the conductor exposed to helium.

The idealized heat transfer characteristic is shown in Fig. 1.

In the region of constant heat transfer coefficient the solution for the terminal characteristics with and without a heat addition q_h per unit length of conductor were derived previously.[2,3] The derivation is relatively simple and is reproduced below for convenience:

The total heat flux per unit of conductor area exposed to liquid helium is made up of that due to ohmic heating plus that due to the heat source of q_h per unit length:

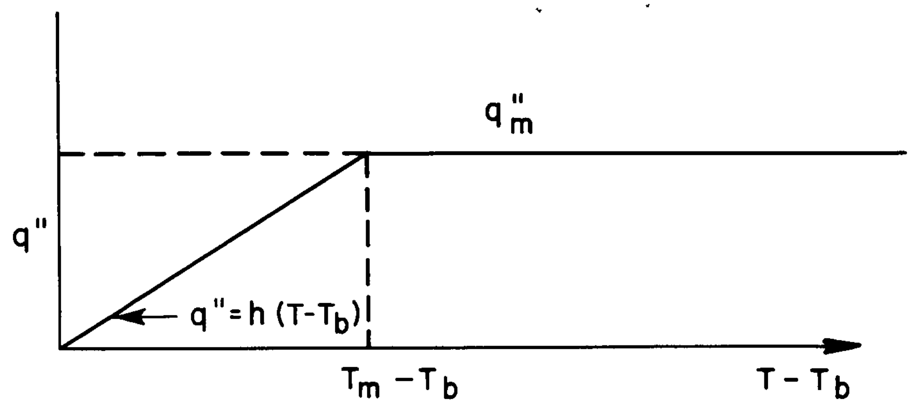

Fig. 1 First Approximation to the Non-linear Heat Transfer Character of Liquid Helium

$$q'' = \frac{vI}{P} + \frac{q_h}{P} \tag{1}$$

where v is the voltage per unit length of conductor, I the total conductor current, and P is the perimeter of the cross section that is exposed to liquid helium.

If we define a parameter f which is the ratio of the current flowing in the normal substrate to the total current:

$$f = \frac{I - I_s}{I} = 1 - \frac{I_s}{I_c}\,\frac{I_c}{I} \tag{2}$$

where I_c is the critical current of the superconductor at the bath temperature and at whatever magnetic field it is exposed to, and I_s is the superconductor current which varies with temperature and magnetic field.

For this part of the analysis assume a constant externally applied magnetic field. The superconductor current I_s is then a function of temperature only and can be approximated by a straight line:

$$\frac{I_s}{I_c} = 1 - \frac{T - T_b}{T_c - T_b} \tag{3}$$

where T is the conductor temperature T_c the critical temperature at zero current while exposed to the magnetic field, and T_b is the bath temperature.

The voltage per unit length of conductor is then:

$$v = \frac{\rho}{A}\ I f = \frac{\rho}{A}\ I_c \left(\frac{I}{I_c}\right) \left[1 - \frac{1 - \frac{T - T_b}{T_c - T_b}}{\frac{I}{I_c}} \right] \tag{4}$$

where ρ/A is the resistance per unit length of the normal substrate.

Introducing the following dimensionless variables:

$$\theta = \frac{T - T_b}{T_c - T_b}$$

$$\tau = \frac{I}{I_c}$$

The equation for heat flux becomes:

$$q'' = \frac{\rho I_c^2}{PA} \tau^2 \left(1 - \frac{1-\theta}{\tau}\right) + \frac{q_h}{P} \tag{5}$$

This equation must now be solved for two regions, the region of constant heat transfer coefficient h, and the region of constant heat flux q''_m.

Constant h: $(T < T_m)$

For constant heat transfer coefficient the relationship between heat flux per unit area and surface temperature rise is:

$$q'' = h(T - T_b) \tag{6}$$

substitution into Eq. (5) and solving for θ yields:

$$\theta = \frac{\alpha\tau(\tau - 1) + Q_{h_o}}{1 - \alpha\tau} \tag{7}$$

where the dimensionless parameters are defined as:

$$Q_{h_o} = \frac{q_h}{hP(T_c - T_b)}$$

$$\alpha = \frac{\rho I_c^2}{hPA(T_c - T_b)}$$

Using Eqs. (2) and (3), the value of f is:

$$f = \frac{\tau - 1 + Q_{h_o}}{\tau(1 - \alpha\tau)} \tag{8}$$

and the voltage per unit length in dimensionless form:

$$V = \frac{vA}{\rho I_c} = f\tau = \frac{\tau - 1 + Q_{h_o}}{1 - \alpha\tau} \tag{9}$$

Constant Heat Flux $(T > T_m)$

For constant heat flux:

$$q'' = q''_m = h\Delta T_m \tag{10}$$

Substitution into Eq. (5) yields:

$$\theta = \frac{\theta_m + \alpha\tau\ (1 - \tau) - Q_{h_o}}{\alpha\tau} \tag{11}$$

Where θ_m is:

$$\theta_m = \frac{T_m - T_b}{T_c - T_b}$$

The value of f can be found using Eqs. (2) and (3):

$$f = \frac{\theta_m - Q_{h_o}}{\alpha\tau^2} \tag{12}$$

and the voltage:

$$V = \frac{vA}{\rho I_c} = f\tau = \frac{\theta_m - Q_{h_o}}{\alpha\tau} \tag{13}$$

<u>Terminal Characteristics</u>

Using the above equations the terminal characteristics of a conductor can be calculated. Figure 2 shows the results for the case of no heating ($Q_{h_o} = 0$) and for $\theta_m = 0.25$.

For $\alpha < 1$, no voltage appears until the current reaches the critical value, ($I/I_c = 1$) the voltage then rises gradually until the limit of nucleate boiling is reached; at this point all the current is expelled suddenly from the superconductor and transferred to the copper. If the current is then reduced, a recovery occurs in which all or most of the current transfers back into the superconductor. (This cycle is shown in Fig. 2 for $\alpha = 0.5$). For $\alpha > 1$, no voltage appears until the current reaches the critical value; then the current transfers abruptly out of the superconductor. Lowering of the current to a recovery value will again result in a return of all the current into the superconductor.

There are three points which characterize the behavior of a particular conductor exposed to a magnetic field.

1. <u>The critical current</u>

   This occurs at $I/I_c = 1.0$ (14)

2. <u>The transition from nucleate boiling to film boiling</u> from the region where current is shared between the superconductor and its substrate.

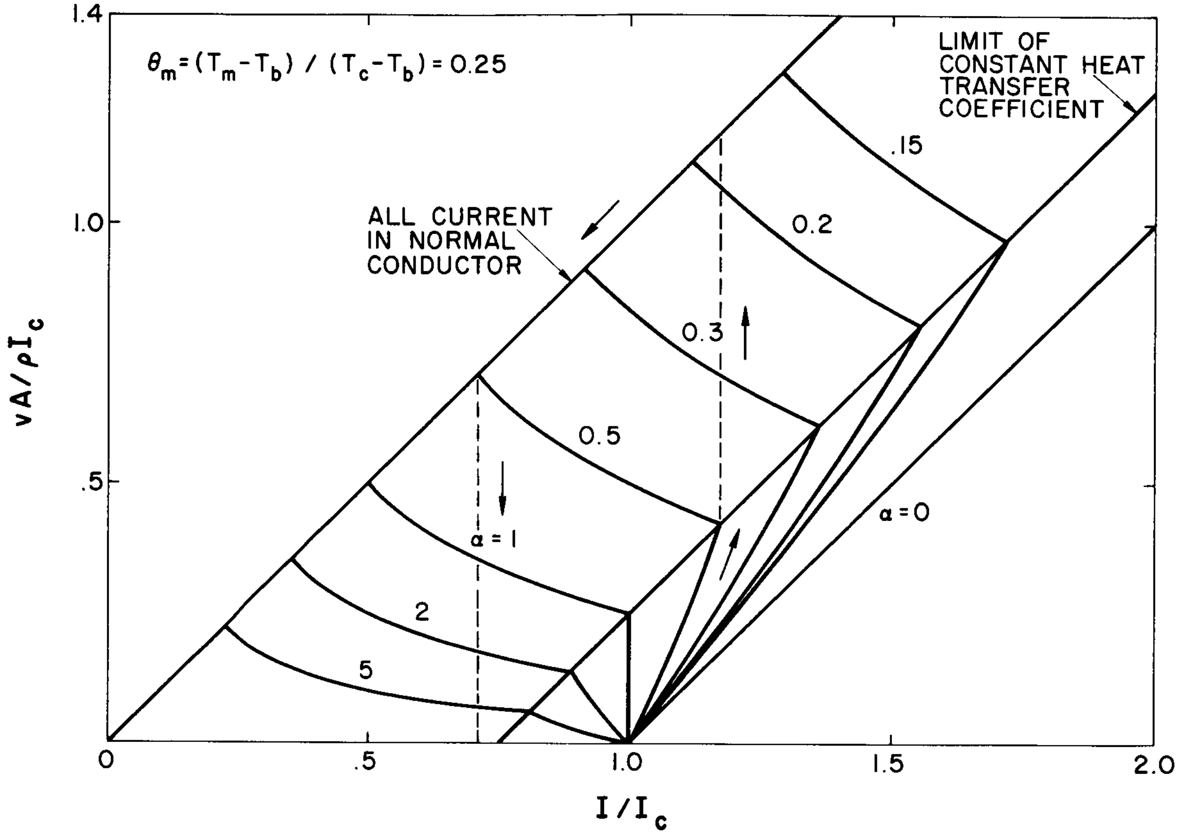

Fig. 2 Dimensionless terminal characteristics of a conductor, taking into account through a simplified model, the change in heat transfer characteristics resulting from a transition from nucleate to film boiling.

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This occurs at a current which is found by equating Eqs. (13) and (9) and solving

$$\frac{I}{I_c} = \frac{1 - \theta_m}{2} + \sqrt{\left(\frac{1 - \theta_m}{2}\right)^2 + \frac{\theta_m - Q_{h_o}}{\alpha}} \tag{15}$$

3. The recovery from film boiling to nucleate boiling with all the current in the normal substrate.

This is simply the f = 1 point, which from Eq. (12) results in:

$$\frac{I}{I_c} = \sqrt{\frac{\theta_m - Q_{h_o}}{\alpha}} \tag{16}$$

Hysteresis in the boiling curve which results in a transition from nucleate to film at a different heat flux than the recovery transition from film to nucleate can be easily taken into account. These two points correspond to the takeoff point, which can be characterized by a value of q''_m and ΔT_m (which determine α and θ_m), and the recovery point which can be characterized by a different value of the same parameters. Operation of a coil never occurs between these two points since the slope of the V-I characteristic is negative, hence, the details of the curve are not important. It can therefore be concluded that the important characteristics of the heat transfer curve are the constant heat transfer curve are the constant heat transfer coefficient part, the limit of nucleate boiling, and the recovery to nucleate boiling.

Behavior Map

A map can now be drawn up of the different regions of operation for a particular conductor. This type of map is shown schematically in Fig. 3. The map must take into account the variation of the critical temperature at zero current T_c and the restivity of the substrate ρ with magnetic field.

The behavior of a typical superconductor has the following regions and curves:

1. An I-H curve - represents the current-carrying capacity of the superconductor at the bath temperature.

2. A recovery curve - defined by the condition for maximum nucleate boiling heat flux, with all the current in the normal substrate; conduction of heat between adjacent turns must be taken into account by using an effective value for the maximum heat flux.

3. The takeoff curve - defined by the condition for maximum nucleate boiling heat flux under conditions of current sharing between the superconductor and the normal substrate. Conduction between turns must also be taken into account in calculation of this curve.

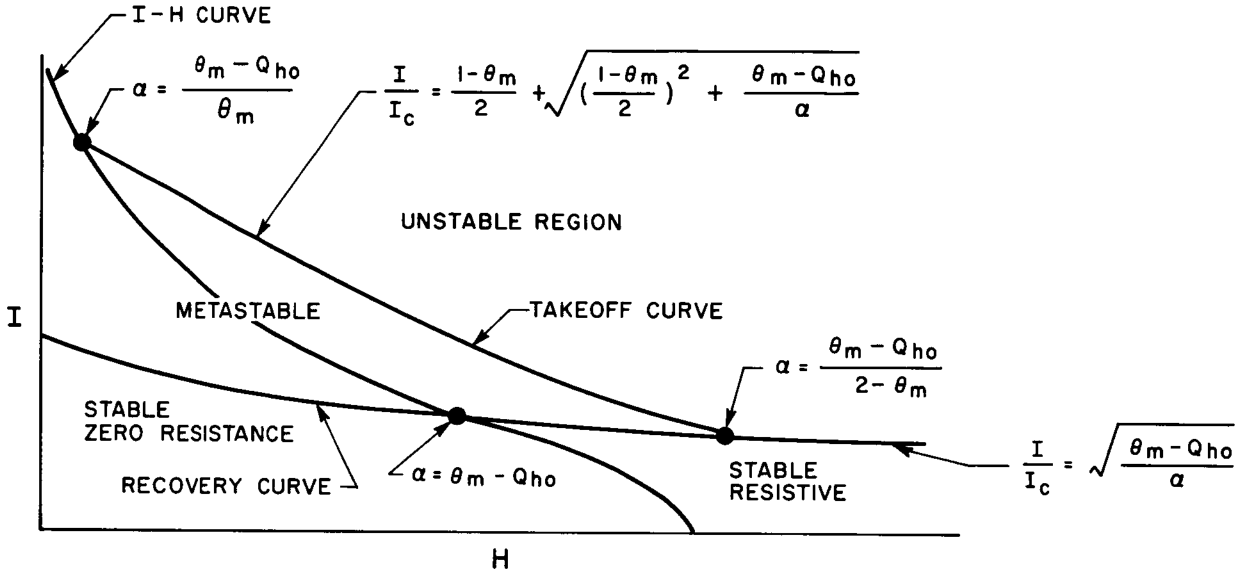

Fig. 3 Map Indicating Modes of Behavior as Predicted by a Zero-Dimensional Analysis Incorporating a Non-linear Heat Transfer Characteristic

4. Stable zero resistance region - below the recovery curve and the I-H curve.

5. Stable resistive region - above the I-H curve and below the recovery curve.

6. Metastable region - between the takeoff and recovery curves; either below or above the I-H curve in which the conductor can be triggered by a disturbance from the fully superconducting region (below the I-H curve) or from the nucleate boiling region (above the I-H curve) into the film boiling region with the simultaneous local transfer of the current from the superconductor into the normal conductor.

7. Fully normal region - operation in this region is possible only with all the current in the normal conductor.

III. COMPOSITE SUPERCONDUCTOR BEHAVIOR INCLUDING GRADIENTS ALONG THE CONDUCTOR AND INCORPORATING A NON-LINEAR HEAT TRANSFER CHARACTERISTIC

The configuration to be considered consists of a long composite conductor consisting of a superconductor in intimate thermal and electrical contact with a substrate which is cooled by a bath at temperature T_b. The entire conductor is immersed in a uniform, known magnetic field. The cross-sectional area is assumed to be small enough and the thermal conductivity high enough so that transverse gradients may be neglected. Longitudinal gradients in temperature will be initiated by a point heat source far from the ends of the conductor. This is essentially the same configuration analyzed under "One-Dimensional Effects" in the first quarterly progress report.[4] In this section, however, the theoretical model is modified and improved by incorporating the non-linear heat transfer characteristic described in the previous section and illustrated in Fig. 1.

Assuming that the current-carrying capacity of the superconductor is given by*:

$$I_s = I_c\left(1 - \frac{T - T_b}{T_c - T_b}\right) = I_c\,(1 - \theta) \tag{17}$$

defining that the fraction of the total current carried by the substrate is:

$$f = 1 - \frac{I_s}{I_c} = 1 - \frac{1 - \theta}{\tau} \tag{18}$$

and remembering the characteristics of the constant heat transfer coefficient analysis done earlier,[4] the sequence of events may be expected to be as shown in Fig. 4. In Fig. 4a, the heat input to the origin is small enough relative to the ability of the bath to cool the conductor through nucleate boiling (constant heat transfer coefficient) so that the dimensionless temperature at the origin, θ_o, is less than θ_m, the temperature corresponding to the transition from nucleate to film boiling, and also that $\theta_o < (1 - \tau)$ (the condition for onset of resistance or $f = 0^+$). Hence, the current will flow entirely in the superconductor. If the heat input is increased as in Fig. 4b, until $\theta_m < \theta_o < (1 - \tau)$, the conductor will remain in a superconducting state, but for $0 < x < x_m$, it will be cooled by film boiling and for $x > x_m$ by nucleate boiling. In Fig. 4c, the heat input is high enough so that $(1 - \tau) < \theta_o < 1$, hence, current is shared for $0 < x < \Delta x$ and joule heating becomes important.

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* Nomenclature will be identical to that in the previous section unless otherwise noted.

For $x > \Delta x$, all the current flows in the superconductor. Cooling is again by film boiling for $0 < x < x_m$ and by nucleate boiling for $x > x_m$. Increasing the heat at the origin until $\theta_o > 1$ is illustrated in Fig. 4d, where all the current flows in the substrate for $0 < x < x_1$, the current transfers back into the superconductor for $x_1 < x < x_1 + \Delta x$ and all the current flows in the superconductor for $x > x_1 + \Delta x$.

If the bath can cool the conductor sufficiently, the transition from Fig. 4a through Fig. 4d and back again as q_{h1} is increased then decreased, may be expected to be controllable and reversible. If, on the other hand, cooling is sufficiently poor, q_{h1} may be increased to some point where the length of the non-superconducting section will suddenly propagate to infinity and recovery to a fully superconducting state is not possible even if q_{h1} is reduced to zero. Between these two extremes is the hysteretic behavior where the heat may be increased to a point where the resistive length increases suddenly but remains finite. A sudden recovery may then be obtained by reducing q_{h1} to some lower value.

In the previous quarterly progress report, conditions were found which bracket the above modes of behavior under the assumption of a constant heat transfer coefficient. This section presents the preliminary results of an attempt to determine these same conditions for the case of a constant heat transfer coefficient for $0 < \theta < \theta_m$ and a maximum allowable heat flux per unit area, q_m, for $\theta > \theta_m$. The analysis is limited to $(1 - \tau) > \theta_m$. Since θ_m is relatively small, however, results will be applicable to one-dimensional situations where the current carried does not approach the critical current at the bath temperature.

To determine a general governing equation, consider the element of length shown in Fig. 5. The equation will be written including terms for both cooling by q_m and by constant h. This equation will then be applied to different sections of the wire shown in Fig. 4c. In each section, certain terms will not apply.

Begin by writing the steady-state energy equation for the element in Fig. 5.

$$W_i A dx - q_m P dx - h\, P dx\, (T - T_b) + kA \frac{d^2T}{dx^2} = 0 \qquad (19)$$

where:

W_i = joule heat per unit volume (when $f \neq 0$)

A = cross-sectional area

P = cooled perimeter

q_m = maximum heat flux (when $\theta > \theta_m$)

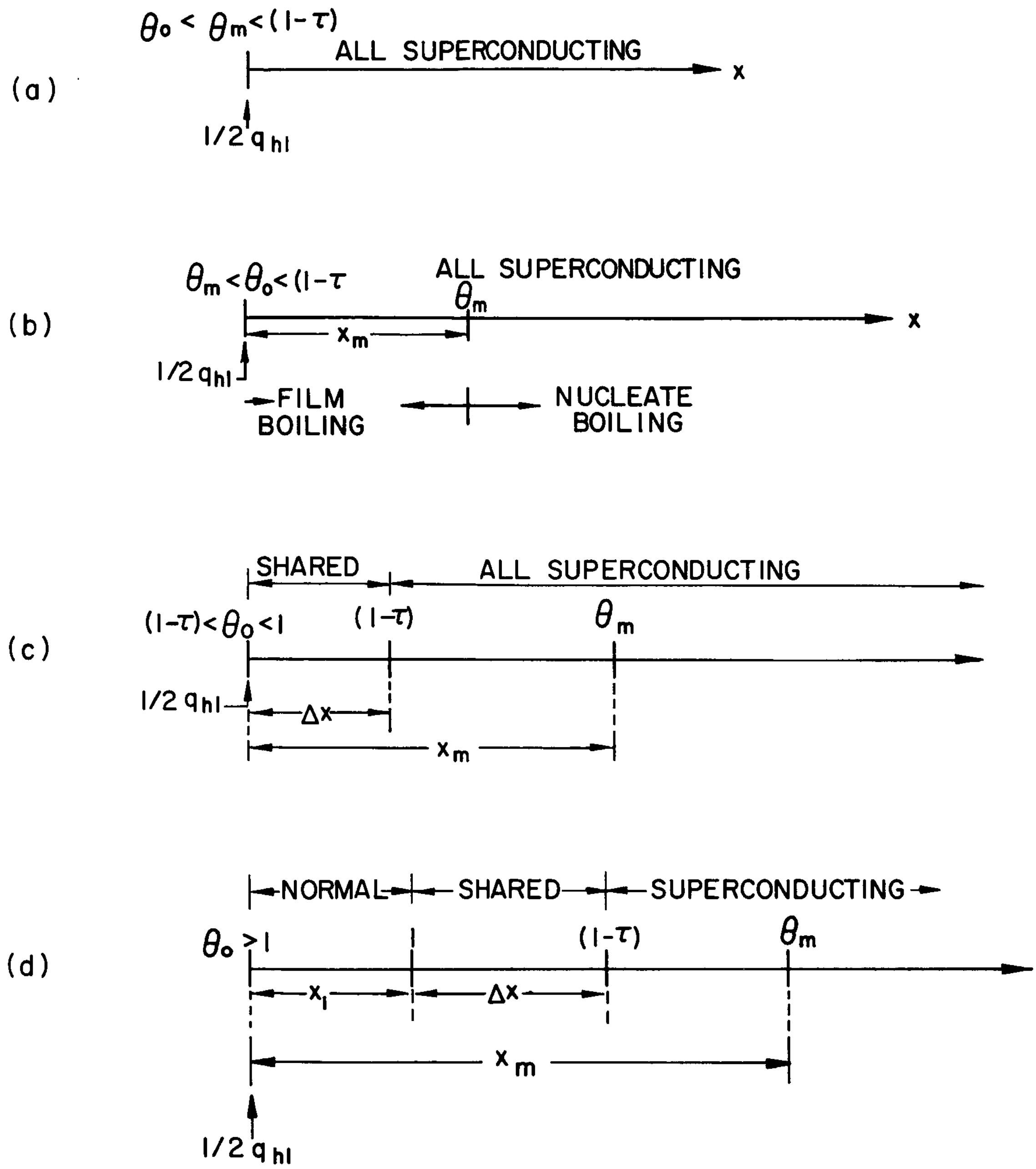
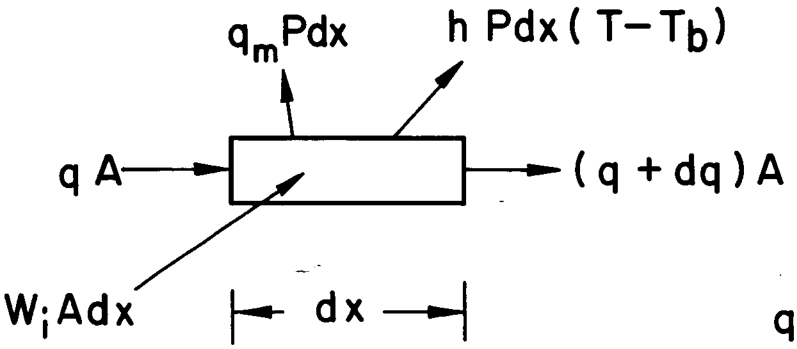


Fig. 4a Conductor Cooled Entirely by Nucleate Boiling

b Conductor Cooled Partially by Nucleate and Partially by Film Boiling

c Cooling after the Onset of Resistance but before all of the Current Transfers into the Substrate at the Origin

d Configuration after $\theta_0 > 1$, Provided $x_1 \not\to \infty$



$$q = - \, kA \frac{dT}{dx}$$

Fig. 5 Element for Determining General Governing Equation

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h = heat transfer coefficient (when $\theta < \theta_m$)

k = thermal conductivity

The voltage per unit length is:

$$v = \frac{\rho f I}{A} \tag{20}$$

hence, the joule heat per unit volume is:

$$W_i = \frac{v \cdot I}{A} = \frac{\rho f I^2}{A^2} \tag{21}$$

If a dimensionless length is defined by:

$$\xi = \frac{x}{\sqrt{\frac{kA}{hP}}}$$

then Eq. (19) may be written as:

$$\frac{d^2\theta}{d\xi^2} - \theta - Q_m + \alpha\tau^2\left(1 - \frac{1-\theta}{\tau}\right) = 0 \tag{22}$$

where

$$Q_m = \frac{q_m}{h(T_c - T_b)}$$

Earlier definitions of θ, α, τ, and f have been utilized.

The first term in Eq. (22) is due to conduction; the second to a constant heat transfer coefficient (nucleate boiling); the third to a maximum heat flux (transition to film boiling); the last to joule heating. Equation (22) may now be applied to the configuration shown in Fig. 4c to determine the governing equations for the problem, which are:

$$\frac{d^2\theta_1}{d\xi^2} - Q_m + \alpha\tau^2\left(1 - \frac{1-\theta_1}{\tau}\right) = 0 \text{ for } 0 < \xi < \Delta\xi\text{, where } 1 > f > 0 \tag{23}$$

$$\frac{d^2\theta_2}{d\xi^2} - Q_m = 0 \text{ for } \Delta\xi < \xi < \xi_m\text{, where } f = 0 \tag{24}$$

$$\frac{d^2\theta_3}{d\xi^2} - \theta_3 = 0 \text{ for } \xi > \xi_m \tag{25}$$

The boundary conditions are that

$$\theta_3 \rightarrow 0 \text{ as } \xi \rightarrow \infty$$

$$\left.\frac{d\theta}{d\xi}\right|_{\xi = 0} = - \frac{q_{h1}}{2\sqrt{hPkA}\,(T_c - T_b)} \equiv -Q_{h1}$$

and that the temperature and its derivative be continuous at $\xi = \xi_m$ and $\xi = \Delta\xi$. Equations (23), (24), and (25), together with the boundary conditions determine the problem and it may be shown that:

for $0 < \xi < \Delta\xi$,

$$\theta_1 = C_1 \sin(\sqrt{a\tau}\,\xi) + C_2 \cos(\sqrt{a\tau}\,\xi) + \frac{Q_m}{a\tau} + (1 - \tau) \tag{26}$$

where:

$$C_1 = - \frac{Q_{h1}}{\sqrt{a\tau}}$$

$$C_2 = \frac{Q_{h1}}{\sqrt{a\tau}} \tan(\sqrt{a\tau}\,\Delta\xi) - \frac{Q_m}{a\tau \cos(\sqrt{a\tau}\,\Delta\xi)}$$

for $\Delta\xi < \xi < \xi_m$

$$\theta_2 = \tfrac{1}{2} Q_m,(\xi^2 - \Delta\xi^2) - (\theta_m + Q_m\,\xi_m)(\xi - \Delta\xi) + (1 - \tau) \tag{27}$$

for $\xi > \xi_m$

$$\theta_3 = \theta_m e^{-(\xi - \xi_m)} \tag{28}$$

The lengths $\Delta\xi$ and ξ_m are determined by:

$$Q_{hl} = \frac{Q_m}{\sqrt{\alpha\tau}} \sin(\sqrt{\alpha\tau}\ \Delta\xi) +$$

$$Q_m \sqrt{\left(\frac{\theta_m}{Q_m}\right)^2 + \frac{2}{Q_m}(1 - \tau - \theta_m)}\ \left[\cos(\sqrt{\alpha\tau}\ \Delta\xi)\right] \tag{29}$$

and

$$\xi_m = \Delta\xi - \frac{\theta_m}{Q_m} + \sqrt{\left(\frac{\theta_m}{Q_m}\right)^2 + \frac{2}{Q_m}(1 - \tau - \theta_m)} \tag{30}$$

Assume that $\theta_m = Q_m$, the case shown in Fig. 1. For this condition the amount of heat required to cause the onset of resistance may be found using Eq. (29) and shown to be:

$$Q_{hl}\Big|_{or} = \sqrt{\theta_m\left[2(1 - \tau) - \theta_m\right]}, \quad (1 - \tau) > \theta_m \tag{31}$$

which is restricted to $(1 - \tau) > \theta_m$. For $(1 - \tau) < \theta_m$, the constant h analysis presented in the first quarterly report[5] is valid and this condition is:

$$Q_{hl}\Big|_{or} = (1 - \tau) \qquad\qquad (1 - \tau) < \theta_m \tag{32}$$

By taking $\frac{\partial Q_{hl}}{\partial \Delta\xi}$ using Eq. (29) and setting the result equal to zero, the length $\Delta\xi_T$ is found which determines the sudden takeoff in voltage, either as an uncontrolled quench or to another operating point as in hysteresis. Carrying out this process shows that the takeoff length is given by:

$$\tan(\sqrt{\alpha\tau}\ \Delta\xi_T) = \left\{\alpha\tau\left[1 + \frac{2}{\theta_m}(1 - \tau - \theta_m)\right]\right\}^{-\frac{1}{2}} \tag{33}$$

The heat input to the origin which corresponds to takeoff, Q_{hlT}, is determined by Eqs. (29) and (33).

$$Q_{h1T} = \sqrt{\theta_m^2 \left(\frac{1}{\alpha \tau} - 1\right) + 2\,\theta_m\,(1 - \tau)} \tag{34}$$

The above equations govern the situation depicted by Fig. 4c, hence, it is necessary that $\theta \leq 1$ at the origin. This inequality, together with Eqs. (26), (33), and (34) yield the condition that:

$$\alpha\,\tau^2 > \theta_m \tag{35}$$

It is, therefore, necessary for Eq. (35) to be satisfied when $Q_{h1} \rightarrow Q_{h1T}$ (and in turn $\Delta\xi \rightarrow \Delta\xi_T$) in order for takeoff to occur from the configuration in Fig. 4c. Hence,

$$\alpha\,\tau^2 < \theta_m \tag{36}$$

represents the limit of stability since takeoff is then not possible. This condition corresponds to the requirement that $\alpha\tau^2 < 1$ for stability to be satisfied when the conductor is cooled via a constant heat transfer coefficient throughout its length.[6]

IV. TWO-DIMENSIONAL ANALYSIS OF THE CHARACTERISTICS OF A COMPOSITE SUPERCONDUCTOR INCLUDING THE EFFECTS OF ANISOTROPIC CONDUCTIVITY AND OF A NON-LINEAR HEAT TRANSFER COEFFICIENT

This particular case is of interest when considering relatively large coil systems in which long lengths of conductor are subjected to essentially the same field and cooling environment, but where, because of constructional detail, the effective thermal conductivity is anisotropic in the plane perpendicular to the winding direction.

The analysis will model the non-linear heat transfer character of the bath in the manner utilized in the previous two sections and illustrated in Fig. 1. A general equation including all the terms in the energy equation will be developed at the outset then applied to different sections of the conductor with cancellation of appropriate terms. An additional external point heat source will be assumed to be located at the origin for the purpose of determining limits of stability. The combination of the two-dimensional geometry together with a point heat source immediately implies that a normal region will always exist about the source since an infinite temperature gradient and, in turn, a mathematically infinite source temperature, will be required to drive a finite amount of heat, q_{h2}.

With reference to Fig. 6, the following steady state energy equation may be written.

$$-k_x \frac{\partial^2 T}{\partial x^2} dy\, dx - k_y \frac{\partial^2 T}{\partial y^2} dy\, dx + \frac{q_m}{P'} dx\, dy + \frac{h}{P'} (T - T_b)\, dx\, dy - W_i\, dx\, dy = 0 \tag{37}$$

where:

k_x = effective thermal conductivity in the x - direction

k_y = effective thermal conductivity in the y - direction

q_m = maximum heat flux per unit area as indicated in Fig. 1

P' = ratio of volume to internal heat transfer surface area

W_i = joule heating per unit volume

h = heat transfer coefficient

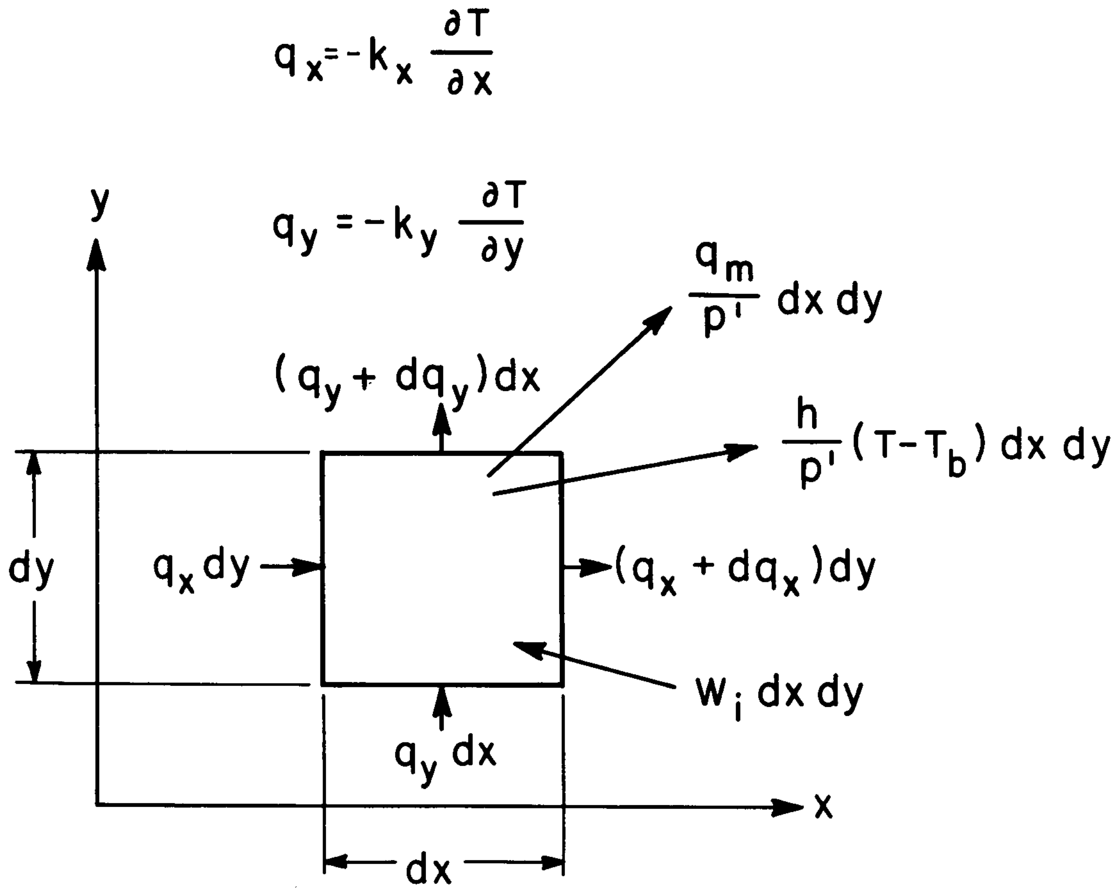

Fig. 6 Energy Balance for Determining the General Equation for the Two-Dimensional Case with Anisotropic Conductivity and Non-linear Heat Transfer Coefficient

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Define:

$$x'^2 = \frac{k}{k_x} x^2$$

$$y'^2 = \frac{k}{k_y} y^2$$

$$r^2 = x'^2 + y'^2$$

and assume $dxdy = dx' \, dy'$

then $k = \sqrt{k_x k_y}$

and Eq. (37) becomes

$$\frac{k}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r}\right) - \frac{q_m}{P'} - \frac{h}{P'} (T - T_b) + W_i = 0 \tag{38}$$

As in the previous section, let

$$W_i = \frac{\rho f I^2}{A^2}$$

where: ρ = resistivity of the substrate

f = fraction of the current carried by the substrate

I = total current carried by the conductor

A = cross-sectional area of the conductor

and

$$\frac{I_s}{I_c} = 1 - \frac{T - T_b}{T_c - T_b} = 1 - \theta$$

where: I_s = current carrying capacity of the superconductor at temperature T

I_c = critical current at the applied field and the bath temperature, T_b

T_c = critical temperature at the applied field and zero current

Also, let

$$\tau = I/I_c$$

$$\xi = \frac{r}{r_o}$$

$$r_o^2 = \frac{kP'}{h}$$

$$Q_m = \frac{q_m}{h\,(T_c - T_b)}$$

$$a = \frac{\rho\, I^2_c\, P'}{hA^2\,(T_c - T_b)}$$

then Eq. (38) becomes:

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \theta}{\partial \xi}\right) - Q_m - \theta + a\tau^2 \left[1 - \frac{1-\theta}{\tau}\right] = 0 \qquad (39)$$

The first term in Eq. (39) arises from the conduction of heat into the volume element; the second from cooling via a maximum heat flux corresponding to the transition from nucleate to film boiling; the third from cooling by nucleate boiling; the fourth from joule heating.

With reference to Fig. 7, this equation may now be applied to the different regions of the superconductor.

for $0 < \xi < \xi_1$,

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \theta_1}{\partial \xi}\right) - Q_m + a\,\tau^2 = 0 \qquad (40)$$

for $\xi_1 < \xi < \xi_1 + \Delta\xi$

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \theta_2}{\partial \xi}\right) + a\tau\,\theta_2 - Q_m - a\tau\,(1-\tau) = 0 \qquad (41)$$

for $\xi_1 + \Delta\xi < \xi < \xi_m$,

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \theta_3}{\partial \xi}\right) - Q_m = 0 \qquad (42)$$

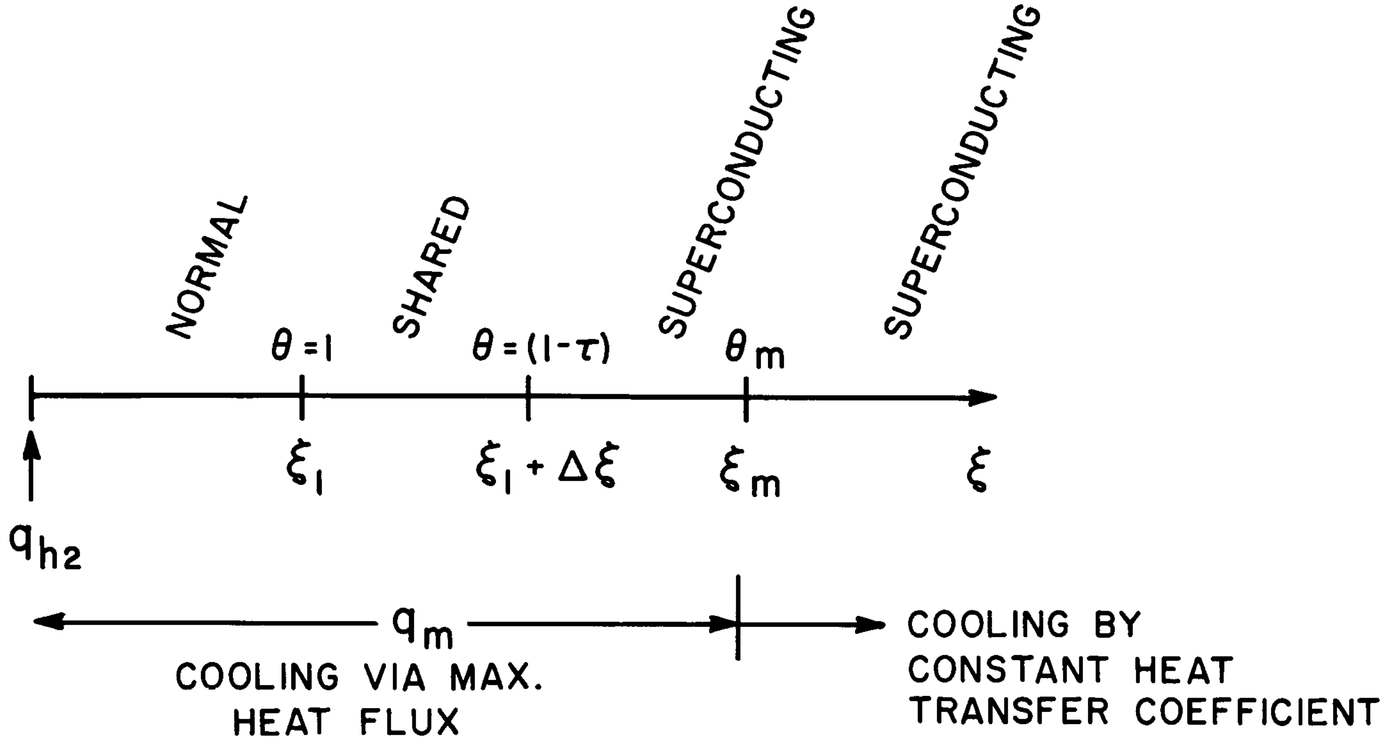

Fig. 7 Regions Denoting Current Distribution for the Two-Dimensional Analysis of Section III

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for $\xi_m < \xi$,

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \theta_4}{\partial \xi}\right) - \theta_4 = 0 \tag{43}$$

In region I, all the current flows in the substrate; in region II, the current is shared between the substrate and superconductor; in regions III and IV, all the current is carried by the superconductor. The temperature in regions I, II, and III is above, θ_m, the temperature corresponding to the transition from nucleate to film boiling; therefore, these regions are constrained by the model (Fig. 1) to be cooled by a given heat flux per unit area. The temperature in region IV is below the transition temperature and is cooled via a constant heat transfer coefficient. A source of heat, q_{h2}, is located at the origin. The analysis is limited to cases where $\theta_m < (1 - \tau)$.

The boundary conditions are that:

$$\theta_4 \rightarrow 0 \text{ as } \xi \rightarrow \infty \tag{44}$$

$$\lim_{\xi \to 0} \xi \frac{d\theta_1}{d\xi} = - \frac{q_{h2}}{2\pi k\,(T_c - T_b)} \equiv - Q_{h2} \tag{45}$$

and that the temperature and its derivatives must be continuous at $\xi = \xi_1$, $\xi = \xi_1 + \Delta\xi$ and at $\xi = \xi_m$.

Using Eqs. (40) to (43), together with the above boundary conditions, it may be shown that:

$$\theta_1 = \frac{1}{4}\,(Q_m - a\,\tau^2)\,\xi^2 - Q_{h2} \ln \xi + C_2, \text{ for } 0<\xi<\xi_1 \tag{46}$$

$$\theta_2 = C_3\, J_o\,(\sqrt{a\tau}\;\xi) + C_4\, Y_o\,(\sqrt{a\tau}\;\xi) + (1-\tau) + \frac{Q_m}{a\tau}, \quad \text{for } \xi_1 < \xi < \xi_1 + \Delta\xi \tag{47}$$

$$\theta_3 = \frac{Q_m}{4}\,\xi^2 + C_5 \ln \xi + C_6, \text{ for } \xi_1 + \Delta\xi < \xi < \xi_m \tag{48}$$

$$\theta_4 = \theta_m \frac{K_o\,(\xi)}{K_o\,(\xi_m)} \text{ for } \xi > \xi_m \tag{49}$$

where:

$J_\nu(x)$ = Bessel function of the first kind of order ν and argument x

$Y_\nu(x)$ = Bessel function of the second kind of order ν and argument x

$K_\nu(x)$ = modified Bessel function of the second kind of order ν and argument x

$$C_2 = 1 + Q_{h2} \ln \xi_1 - \frac{1}{4} (Q_m - a\tau^2) \xi_1^2$$

$$C_3 = \frac{\pi b}{2} \left[B\, Y_o(b) - C\, Y_1(b) \right]$$

$$C_4 = \frac{\pi a}{2} \left[D\, J_1(a) - A\, J_o(a) \right]$$

$$a = \sqrt{a\tau}\ \xi_1$$

$$b = \sqrt{a\tau}\ (\xi_1 + \Delta\xi)$$

$$A = \frac{1}{\sqrt{a\tau}} \left[\frac{Q_{h2}}{\xi_1} - \frac{1}{2} \xi_1 (Q_m - a\tau^2) \right]$$

$$B = -\frac{1}{\sqrt{a\tau}} \left[\frac{Q_m}{2} (\xi_1 + \Delta\xi) + \frac{C_5}{(\xi_1 + \Delta\xi)} \right]$$

$$C = \frac{Q_m}{a\tau}$$

$$D = \tau - \frac{Q_m}{a\tau}$$

$$C_5 = \left\{ 1 - \tau - \theta_m + \frac{Q_m}{4} \left[\xi_m^2 - (\xi_1 + \Delta\xi)^2 \right] \right\} \left(\ln \frac{\xi_1 + \Delta\xi}{\xi_m} \right)^{-1}$$

$$C_6 = 1 - \tau - \frac{Q_m}{4} (\xi_1 + \Delta\xi)^2 - C_5 \ln (\xi_1 + \Delta\xi)$$

The unknown lengths ξ_1, $\xi_1 + \Delta\xi$ and ξ_m are determined by:

$$\frac{\pi b}{2} \left[B\, Y_o (b) - C\, Y_1 (b) \right] J_o (a) + \frac{\pi a}{2} \left[D\, J_1 (a) - A\, J_o (a) \right] Y_o (a) = \tau - \frac{Q_m}{\alpha \tau} \quad (50a)$$

$$\frac{\pi b}{2} \left[B\, Y_o (b) - C\, Y_1 (b) \right] J_o (b) + \frac{\pi a}{2} \left[D\, J_1 (a) - A\, J_o (a) \right] Y_o (b) = \frac{Q_m}{\alpha \tau} \quad (51a)$$

$$\frac{Q_m}{2} \xi_m + \frac{C_5}{\xi_m} = - \theta_m \frac{K_1 (\xi_m)}{K_o(\xi_m)} \quad (52a)$$

For clarity, these equations may be written in a corresponding functional representation which indicates the dependence of ξ_1, $\xi_1 + \Delta\xi$, and ξ_m on α, τ, Q_m, θ_m, and Q_{h2}.

$$f_1 (\xi_1, \xi_1 + \Delta\xi, \xi_m, \alpha, \tau, Q_m, \theta_m, Q_{h2}) = 0 \quad (50b)$$

$$f_2 (\xi_1, \xi_1 + \Delta\xi, \xi_m, \alpha, \tau, Q_m, \theta_m, Q_{h2}) = 0 \quad (51b)$$

$$f_3 (\xi_1 + \Delta\xi, \xi_m, \tau, \theta_m, Q_m) = 0 \quad (52b)$$

These three equations may now, in principle, be used in a manner similar to that utilized in earlier analyses for a determination of the conditions limiting stability. The feasibility and practicality of accomplishing this next step is now under consideration.

V. PRELIMINARY EXPERIMENTAL RESULTS REGARDING ONE-DIMENSIONAL STABILITY CHARACTERISTICS

A series of experiments have been conducted in an effort to exhibit the stability characteristics predicted by the one-dimensional theory. In the following, the test facility is described and typical test results are included. The qualitative agreement with the predictions of the theory is obvious, however, data reduction is not yet complete so that a quantitative comparison is not included.

Experimental Apparatus

A test facility which closely approximates the model underlying the one-dimensional analyses must necessarily incorporate a relatively long composite conductor immersed in a magnetic field and heat transfer environment which are invariant with length. It should also include a small heat source which can introduce a perturbation of known size into the system. An experiment which fulfilled these requirements was carried out with the apparatus illustrated in Fig. 8.

Tests were conducted on two different composite conductor samples: (1) a 0.02" OD - 0.01" core, copper clad Nb Ti and (2) a 0.016" OD - 0.01" copper clad Nb Ti. Each sample was approximately 40" long and would on a grooved phenolic sample holder as shown in Fig. 8. The ends of the sample were soldered to copper end plates which were the terminals of the sample current leads. A 0.003" D. nichrome wire was wound bifilar with the sample as shown in the insert. Voltage was measured across the sample utilizing the nichrome wire as the voltage tap to the lower end of the superconductor. Induced voltages arising from any high frequency fluctuation of the essentially steady applied magnetic field were thus cancelled. Voltage taps were located two inches from the sample ends, well away from the joint to the sample current leads. A uniform field was provided during the tests by a relatively long three inch bore superconducting solenoid.

Typical Test Results

Typical data is shown in Fig. 9 which consists of four plots with heater current (proportional to $\sqrt{q_{h1}}$) as abscissa and dample voltage as ordinate. All runs were taken at the same value of magnetic field, hence, I_c was fixed and α is the same in all cases. Each of the four runs was taken at constant sample current, I, and, therefore constant τ. Figure 9 indicates the effect of increasing τ on stability. To aid qualitative interpretation of these results, Fig. 6 from the First Quarterly Progress Report[7] is repeated in this report as Fig. 10. It should be noted that the theoretical results in Fig. 10 are for a given $\alpha = 3$ which does <u>not</u> correspond to the α

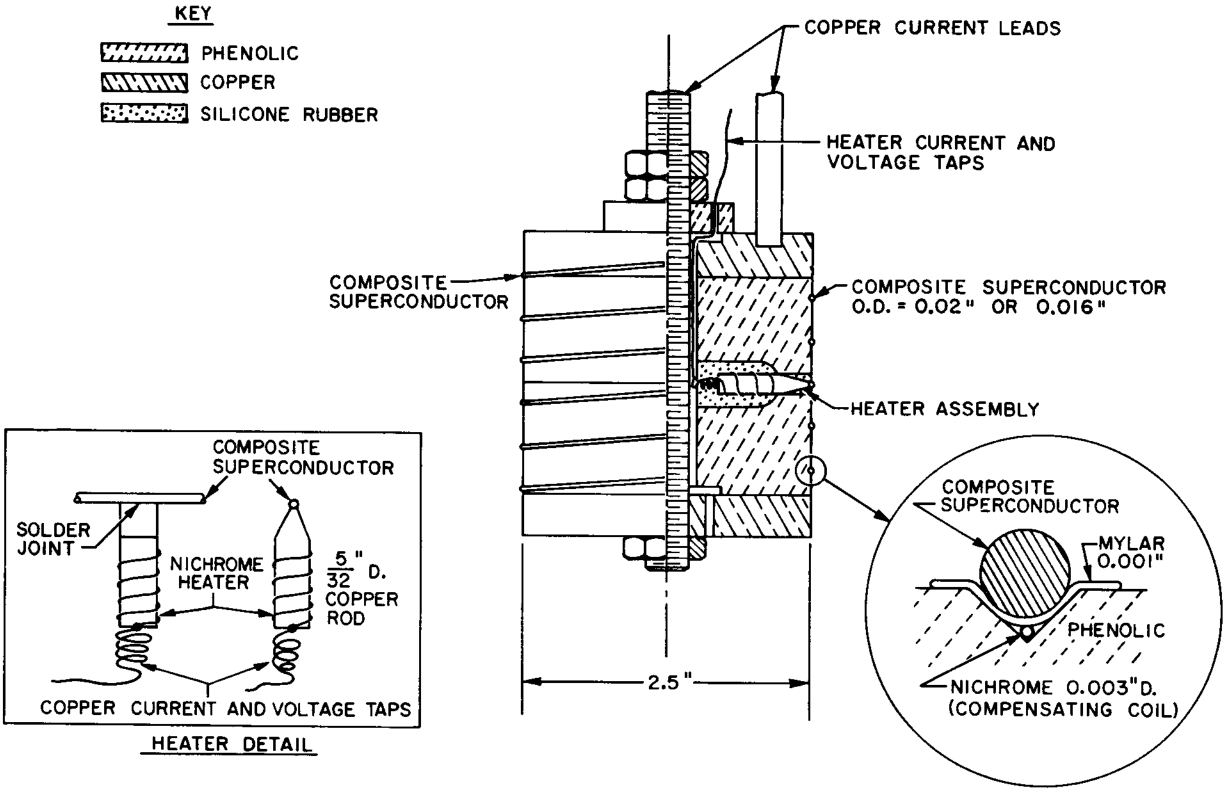

Fig. 8 Test Apparatus for Correlation of One-Dimensional Effects

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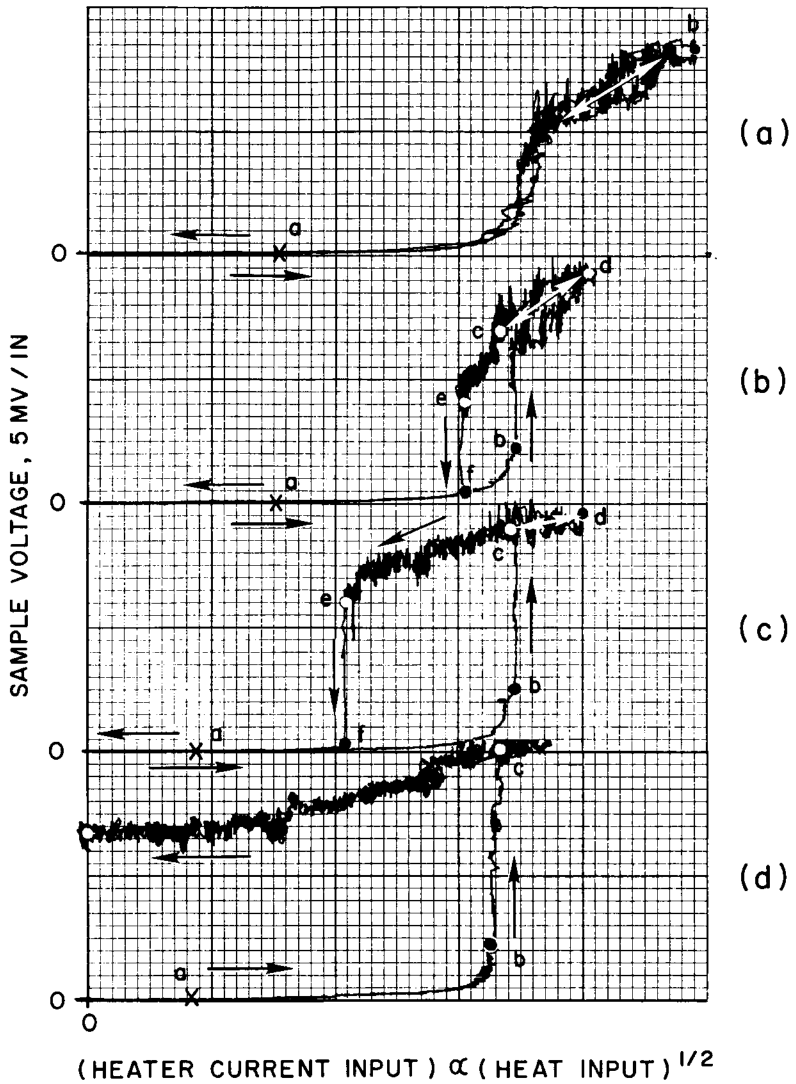

Fig. 9 Typical Test Results Regarding One Dimensional Stability

a) $\frac{I}{I_c}$ = .88, stable operation

b) $\frac{I}{I_c}$ = .91, hysteretic behavior

c) $\frac{I}{I_c}$ = .93, hysteretic behavior

d) $\frac{I}{I_c}$ = .94, unstable operation

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in the experiment. In this report, however, interest is centered on qualitative comparison so that the use of analytical results at constant α for comparison with experimental results at a constant but different α, is justified.

The scale on sample voltage in Fig. 9 is 5 mv/in. Values of heater current at which the onset of resistance was detected are indicated as point a. These values were determined by repeating the runs as far as the onset of resistance and using a scale of 0.2 mv/in. for sample voltage.

Figure 9a illustrates stability analogous to operation in the stable region of Fig. 10. As heater power is increased from zero to a maximum at point b then decreased, operation is controlled and reversible throughout. Note that heat may be increased from zero to a, before resistance is exhibited.

Figure 9b is a run at a slightly increased value of τ and corresponds to operation in the hysteretic "a" region in Fig. 10. Heat input was increased from zero. At a, the onset of resistance occurred. Voltage then increased controllably and continuously until at point b (corresponding to the take-off value of the heat input, Q_{h1T}) a discontinuous rise in voltage takes place up to a steady state operating point at c. Operation is reversible between c and d and back to c. A very slow decrease in heater power from c yields a continuous decrease in voltage until at point e (corresponding to the recovery value of the heat input Q_{h1R} in Fig. 10) a discontinuous decrease in sample voltage occurs to an operating point with positive resistance at f. Further decrease in heat yields continuous decrease in voltage to zero at a.

Figure 9c also exhibits hysteretic behavior with corresponding points labelled as in Fig. 9b. This run, however, is at a value of τ high that that in Fig. 9b, hence, the size of the hysteresis loop is larger. This is in agreement with Fig. 10 which indicates a greater difference between Q_{h1T} and Q_{h1R} as τ is increased in the hysteretic region.

Figure 9d illustrates unstable behavior. Voltage increases in a controlled and reversible manner with heat input until, at point b, a discontinuous rise in voltage occurs to point c. Decrease in heat input to zero at point g is continuous and no recovery is exhibited. This corresponds to operation in the unstable region of Fig. 10 where voltage increased controllably until the heat input reaches Q_{h1T} where the resistive region propagates to infinity and the sample voltage becomes infinite. The finite sample voltage at c in Fig. 9d arises only from the finite length of the sample. Instability is indicated experimentally by not exhibiting recovery even though the perturbation in the form of the external heat source is removed.

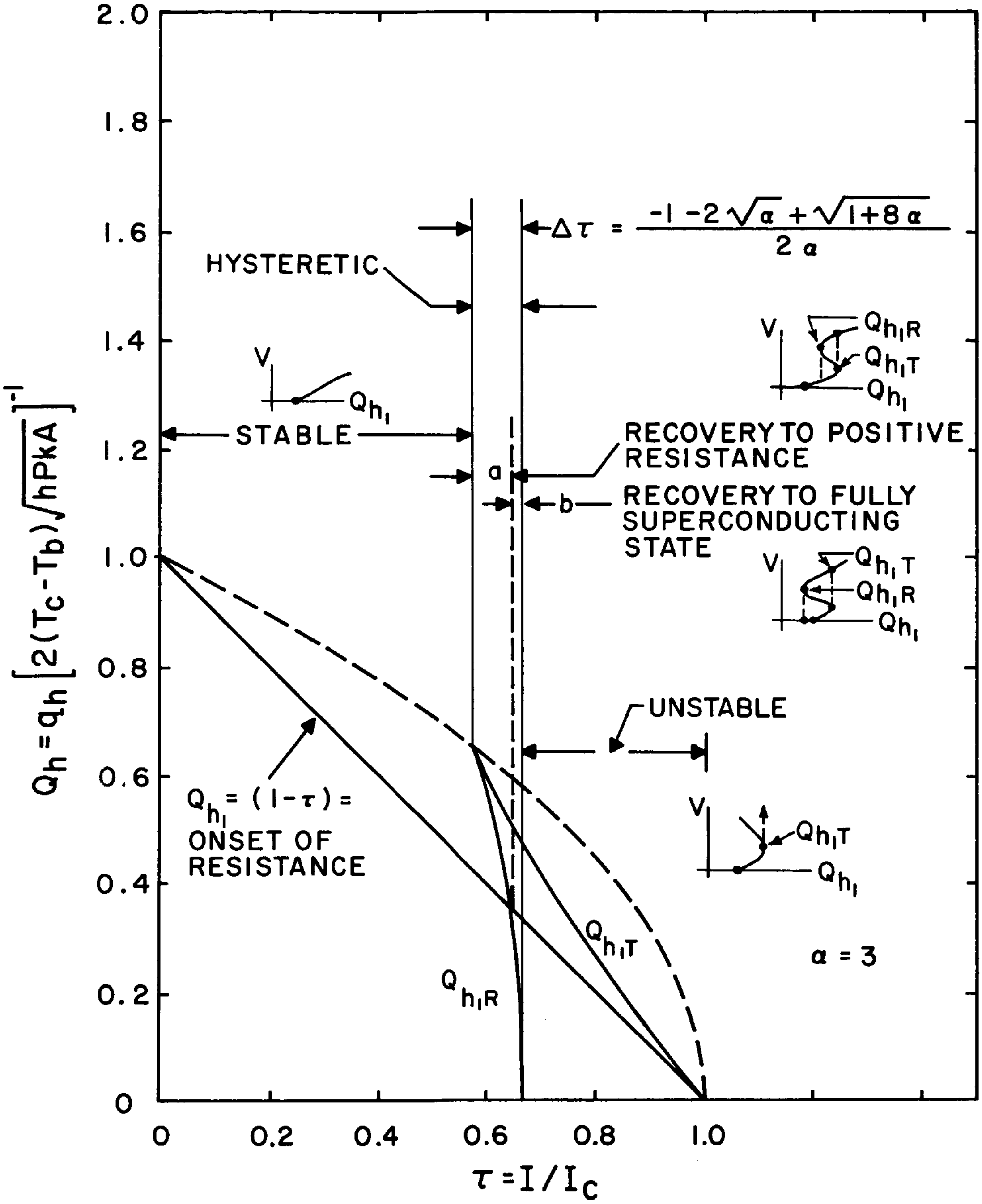

Fig. 10 Heat Input vs Current: An Indicator of the Type of Behavior for Different τ as α is Held Fixed

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VI. CONCLUSION

The previous sections indicate steps along the path leading to a better understanding of the operation of high field supercondicting coils and to methods of predicting coil performance accurately from information regarding short sample terminal characteristics and heat transfer environment.

Analyses have been initiated which model the non-linear behavior of a liquid helium bath with respect to heat flux and experiments have been conducted which are in qualitative agreement with an earlier analysis. In the next quarter, quantitative data reduction will continue with the aim of indicating those areas in the developing theory which require extension and refinement.

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